

Spindown of Isolated Neutron Stars: Gravitational Waves or Magnetic Braking?

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ABSTRACT

We study the spindown of isolated neutron stars from initially rapid rotation rates, driven by two factors: (i) gravitational wave emission due to r-modes and (ii) magnetic braking. In the context of isolated neutron stars, we present the first study including self-consistently the magnetic damping of r-modes in the spin evolution. We track the spin evolution employing the RNS code, which accounts for the rotating structure of neutron stars for various equations of state. We find that, despite the strong damping due to the magnetic field, r-modes alter the braking rate from pure magnetic braking for $B \leq 10^{13}$ G. For realistic values of the saturation amplitude α_{sat} , the r-mode can also decrease the time to reach the threshold central density for quark deconfinement. Within a phenomenological model, we assess the gravitational waveform that would result from r-mode driven spindown of a magnetized neutron star. To contrast with the persistent signal during the spindown phase, we also present a preliminary estimate of the transient gravitational wave signal from an explosive quark-hadron phase transition, which can be a signal for the deconfinement of quarks inside neutron stars.

1. Introduction

Neutron stars are highly compact stars of typical radius $R \sim 12$ km and mass $M \sim 1.5M_\odot$ made mostly of degenerate neutron-rich matter at densities up to several times nuclear matter saturation density $\rho_0 = 2.5 \times 10^{14}$ g/cc. By tracking their long-term thermal and rotational evolution, we can learn about the nature of matter under the crust. For example, Page et al. (2011) have proposed that the thermal history of the neutron star in Cas A may be indicating the recent onset of neutron superfluidity deep in its interior. Recently, Negreiros et al. (2011) have shown how rotational evolution is linked to a reorganization of particle composition in the stellar interior, leading to switching on of exotic neutrino emission processes. A neutron star is a complex system with intertwined physical properties that can change on relatively short astrophysical timescales.

In this paper, we will be concerned with the rotational evolution of a highly magnetized isolated neutron star. The electromagnetic emissions of a neutron star derive from its rotational kinetic energy, and its spindown is usually measured in terms of a braking index, n , which is dependent on the magnetic field configuration ($n=3$ for a dipolar field). Neutron stars can also spindown through gravitational wave emissions associated to the r-mode (Andersson 1998). In fact, the observational interest in r-modes comes from the fact that no neutron stars have been found to spin at rates near the maximum allowed frequency (the “break-up frequency”). r-modes offer a possible explanation of this fact: in rotating neutron stars, these modes lose energy through gravitational waves, which carry away angular momentum from the star and act as braking radiation. As the star spins down, its central density increases. This increase could be sufficient to make baryonic matter undergo phase transitions to more exotic phases of strongly interacting matter, such as quark matter, with possible implications for gamma-ray bursts (Berezhiani et al. 2003; Haensel & Zdunik 2003; Drago et al. 2008) and the formation of quark stars, if theoretical conjectures about the absolute stability of strange quark matter are realized in nature (Itoh 1970; Bodmer 1971; Witten 1984). While it is almost certain that this quark phase, if it exists inside neutron stars, cannot be a free gas of quarks (Özel et al. 2010; Weissenborn et al. 2011), an interacting phase of quarks still appears to be consistent with the recent finding of a $2M_\odot$ neutron star (Demorest et al. 2010).

The main questions we seek to answer are: given a few initial parameters of the newly born neutron star, such as its spin period, magnetic field and baryonic mass, can we determine which neutron stars are likely to eventually manifest a quark phase in their interior? If so, how long before the transition to quark matter occurs? In this work, we take a step towards answering these questions by taking a closer look at the rotational evolution of a newly-born hot neutron star as it spins down, and we focus on two main driving factors - magnetic braking and gravitational radiation from r-modes. We consider different equations of state (EoS) for neutron stars, since the EoS at high density is uncertain and lacking strong empirical constraints (although a recent statistical analysis shows that the equation of state stiffens at high densities and is consistent with the expected range in certain nuclear parameters; Steiner et al. 2010). Furthermore, second generation gravitational wave detectors such as Advanced LIGO will soon be operational, and our work provides an update for a similar theoretical study (Ho & Lai 2000) performed several years ago with regard to the LIGO detector. We should note here that in contrast to the assumptions

in Ho & Lai (2000), our work includes the effects of magnetic damping of r-modes in the spindown evolution, which leads to quantitatively different results.

In section 2, we outline the approach and main equations that describe the neutron star’s spindown. The theoretical analysis follows in part the works of Ho & Lai (2000), as well as Cuofano & Drago (2010) and Rezzolla et al. (2001b). Section 3 collects our main conclusions from this analysis. In section 4, we analyze the evolution of the gravitational wave frequency associated to the growing r-mode, along the lines of Owen et al. (1998), and present results in section 5. We conclude in section 6 with a preliminary calculation of the gravitational wave signal from an explosive deconfinement transition (the “Quark-Nova”).

2. Spindown to deconfinement

In a previous paper (Staff et al. 2006), we had determined birth parameters of a neutron star which would support the transition to deconfinement driven solely by magnetic braking. We found that neutron stars that are born with mass $M \gtrsim 1.5M_{\odot}$ and spin-period $P \lesssim 3$ ms are the best candidates to reach deconfinement density in their core (assumed to be $\sim 5\rho_0$), given a range of neutron star magnetic fields 10^{12} - 10^{15} Gauss. This small value of the initial spin-period (or large spin frequency) is required since magnetic braking from a rotating magnetic dipole is itself proportional to the third power of the spin frequency. Most of the increase in central density occurs soon after the rapidly rotating neutron star is born.

In that work, we neglected the role of gravitational radiation from r-modes in an ad hoc way - by assuming that neutrons stars are born axisymmetric and remain so. In reality, non-axisymmetric perturbations are expected due to the violent process of a supernova, which leaves the new-born neutron star in a turbulent state (Keil et al. 1996; Akiyama & Wheeler 2006). What is then the role of r-modes and gravitational braking in comparison to magnetic braking as far as the time to deconfinement is concerned? Previous studies on the interplay of r-modes and neutron star magnetic fields have focused on different questions - for example, Lee (2009) has studied the r-mode in magnetized neutron stars with $B \leq 10^{12}$ G with an aim to understand X-ray pulsations from local hot spots in accretion-powered pulsars. Ho & Lai (2000) have shown that magnetic fields of $B \geq 10^{14}$ G can make magnetic braking as important as r-mode spindown, specially for slowly rotating stars, and that Alfvén wave driving of the r-mode can also play a role.

Meanwhile, other works (Rezzolla et al. 2001a,b; Kiuchi et al. 2011; Cuofano & Drago 2010) have focused on the evolution of toroidal magnetic fields generated by the secular r-mode. Essentially, the toroidal field is generated by differential rotation of the stellar fluid associated with the r-mode. The differential rotation is a key feature of the r-mode instability in the non-linear regime (Levin & Ushomirsky 2001). Rezzolla et al. (2001a) have shown that, for isolated neutron stars, this effect can amplify existing magnetic fields by two orders of magnitude within the time taken by the r-mode to saturate (\approx few hundred secs.). Back-reaction on the r-mode due to this toroidal field implies that there is an associated magnetic damping, which has to be factored into the spin

evolution. For neutron stars that are accreting from a binary companion, the build-up of toroidal fields to sizeable values is rather slow, and according to Cuofano & Drago (2010), can take up to several hundred years. However, we are interested in this effect on isolated neutron stars. The magnitude of the magnetic damping term F_m is proportional to the integrated time evolution of $\alpha^2(t)$ (Rezzolla et al. 2001b), with α being the r-mode amplitude. These authors obtained a strong damping effect by assuming that the background evolution of $\alpha(t)$ is identical to that given by the absence of any magnetic fields. However, we solve the evolution equations for the star's angular frequency Ω and $\alpha(t)$ with magnetic fields and magnetic damping from the start. F_m grows while the mode is unstable, and once it saturates F_m remains constant. This consistent inclusion of F_m leads to a slower evolutionary path for $\alpha(t)$ towards saturation, hence the magnetic damping is not as effective in suppressing the r-mode. Consequently, for a range of magnetic fields and small r-mode saturation values, we find that this timescale can be much larger than the typical time taken by the neutron star to spindown to typical quark deconfinement density. Therefore, our results clearly indicate that the r-mode, for realistic saturation values, quantitatively affects the spindown evolution of an isolated neutron star. We now discuss the relevant equations for the evolution of the star's rotation rate and r-mode evolution.

The spindown $\dot{\Omega}$ of a neutron star (mass M , radius R) is accompanied by a loss of energy as well as angular momentum. For the radiating star, conservation of total angular momentum $J_{\text{tot}} = J_{\text{star}} + (1 - K_j)J_c$ with K_j a dimensionless constant and $J_{\text{star}} = I\Omega$ (where I is the moment of inertia about the rotation axis) yields (Wagoner 2002; Cuofano & Drago 2010)

$$\frac{dJ_{\text{tot}}}{dt} = 2J_c F_g + \dot{J}_a - I\Omega F_{\text{mag}}. \quad (1)$$

$J_c = -K_c \alpha^2 J_{\text{star}}$ is the canonical angular momentum of the r-mode to 1st order in Ω (Friedman & Schutz 1978). F_g is the rate of gravitational radiation associated to the $l=m=2$ current multipole taken for an $n=1$ polytropic star from eqn.(65) of Andersson et al. (2001). This is expected to be a good approximation to a large part of the neutron star interior. \dot{J}_a is the accretion rate onto the star (assumed zero for our case since we consider isolated stars only) and F_{mag} is the magnetic braking rate (Manchester & Taylor 1977). The dimensionless quantity $K_c = 3\bar{J}/2\bar{I}$ is defined from

$$\begin{aligned} \bar{J} &= \frac{1}{MR^4} \int_0^R \rho r^6 dr, \\ \bar{I} &= \frac{8\pi}{3MR^2} \int_0^R \rho r^4 dr. \end{aligned} \quad (2)$$

where $\rho = \rho(r)$ is the density profile (assumed radially symmetric) of the star¹. J_c evolves in time as a result of competing influences from gravitational damping (which feeds the r-mode),

¹ In principle, equatorial flattening due to rotation renders the density profile asymmetric, but this does not change the value of K_c significantly, which is the basis for our approximation of a symmetric profile.

viscosity (which damps the r-mode) and the magnetic damping term F_m . Both bulk and shear viscosities are included in our analysis although for simplicity we keep the star at a uniform temperature of 10^9K for the duration of the evolution. Although viscosities (especially bulk viscosity) is strongly temperature-dependent, our approximation is not as drastic as it may seem. A typical cooling profile of a neutron star, driven by neutrino emission from the modified URCA process is given by (Owen et al. 1998)

$$T_9(t) = \left(\frac{t}{\tau_c} + T_{i,9}^{-6} \right)^{-1/6}, \quad (3)$$

where T_9 is the star's core temperature T in units of 10^9K , $T_{i,9}$ the initial temperature, and $\tau_c \approx 1\text{ yr}$ is the characteristic cooling time from the modified URCA process. If we begin with birth temperatures $T \sim 10^{11}\text{K}$, we see that within a few seconds, we have $T \sim 10^9\text{K}$. Since the r-mode does not have a large impact until $t \geq 10^2$ seconds, we can approximate $T_9 = T/(10^9\text{K}) \sim 1$. Subsequent cooling is on the timescale of years and is also a small perturbation on our results. Using the evolution equation for J_c , (see eqn.(4) of Cuofano & Drago 2010), one can write

$$\begin{aligned} \frac{\dot{\Omega}}{\Omega} &= -2\alpha^2 K_c [K_j F_g + (1 - K_j)[F_v + F_m]] - F_{\text{mag}} \quad \text{and} \\ \frac{\dot{\alpha}}{\alpha} &= [F_g - [F_v + F_m]] - \frac{\dot{\Omega}}{2\Omega}, \end{aligned} \quad (4)$$

where F_v is the viscous damping rate (Andersson et al. 2001). We solve the two coupled equations above numerically. We use the RNS code (Stergioulas & Friedman 1995) to construct 2-dimensional models of rapidly rotating neutron stars. For a given EoS and for a fixed baryonic mass, the RNS code outputs a sequence of neutron star models (with specific gravitational mass, radius, spin frequency etc.) that have increasing central density and decreasing angular velocity. The fastest model in such a sequence spins at or near Kepler frequency. Note that the magnetic field does not appear in the RNS code, and in any case, its effect on structure at the field values considered here is negligible. The magnetic field only determines the time to deconfinement, unless the spindown is completely r-mode dominated. As in Staff et al. (2006) we assume $5\rho_0$ to be a critical density at which quarks in the interior of the neutron star become deconfined - this is not a well determined number, and may span a range from $(4-8)\rho_0$ if a mixed phase of quark and nuclear matter is favored (Glendenning 1992). To counter this uncertainty, we have checked our numerical results for a higher putative deconfinement density ($\rho \sim 8\rho_0$) and found that it does not change any of our quantitative conclusions by a significant amount.

Using the RNS code, we construct sequences of stars (for a given EoS) with constant baryonic mass and decreasing spin such that the non-rotating configuration has a central density equal to $5\rho_0$. We then calculate the time the rotating star takes until its central density is within 99% of $5\rho_0$. This is a practical way of using eqn.(4) to obtain the minimum mass required (for a given EoS) to support deconfinement, since the time taken to reach zero spin is infinite from the magnetic

braking being proportional to Ω . Since we are interested in the time it takes for a star to spindown to the critical density for deconfinement, by choosing a sequence where the non-rotating model has a central density equal to the critical density, this time gives us the maximum time the star takes to reach the deconfinement density.

We assume a very small initial amplitude of the r-mode for the fastest spinning star in a sequence, with typical initial $\alpha \sim 10^{-6}$, although our results for those cases where the r-mode saturates are insensitive to this initial value. For the case when the mode does not saturate, we do find mild sensitivity to the initial value of α . From the first of eqn.(4), we can calculate the time-step between two consecutive neutron star models in the spin-sequence output by RNS code

$$\Delta t \equiv t_{i+1} - t_i = \frac{\Omega(t_{i+1}) - \Omega(t_i)}{-2K_c\Omega(t_i)\alpha(t_i)^2[K_j F_g + (1 - K_j)(F_v + F_m)] - \Omega(t_i)F_{\text{mag}}} . \quad (5)$$

The inverse dependence on α implies that for small magnetic fields, where F_{mag} is small, Δt is large when the mode amplitude is just starting to grow. The RNS code then needs to generate two widely separated rotating configurations within a sequence, leading to low resolution in some parts of the P vs t curves for small magnetic fields (eg., the curve for $B=10^{12}$ G in Fig.1). However, this is only a problem initially and does not affect the total time to deconfinement. The above equation, along with the second of eqn.(4) is then used to determine the time evolution of α . For our analysis we have used four different magnetic field strengths equally spaced on a logarithmic scale from 10^{12} G to 10^{15} G. We display results for three different equations of state: EoS A which is composed of only neutrons (Pandharipande 1971) and uses a variational principle to determine the minimum energy state; EoS BBB2 which includes muons and uses a field-theoretic many-body approach (Baldo et al. 1997); and finally EoS APR which is relatively stiff and admits a mixed phase of quarks and nuclear matter for the heaviest stars (Akmal et al. 1998). All three EoS generate stable configurations for central densities exceeding $5\rho_0$ and can go up to even $8\rho_0$. They differ in the details of the density profile, with the softest (EoS A) having higher interior densities for the same mass (more compact). The maximum gravitational mass exceeds $2M_\odot$ only for the APR EoS (Akmal et al. 1998), with BBB2 providing a maximum mass of $1.92M_\odot$ (Haensel 2003). EoS A is too soft to generate a $2M_\odot$ neutron star and may seem inadequate to explain recent observations of such massive compact stars (Demorest et al. 2010). However, if such massive stars are really quark or hybrid stars, we cannot rule out EoS A in this way, and its inclusion is still useful simply to examine the trend of a relatively soft EoS on the gravitational signal from r-mode driven spindown.

3. Results: r-mode spindown versus magnetic braking

The following results are obtained on solving eqns.(4) numerically. In Fig. 1, the curves show how the period, starting from the Kepler rotation rate at birth, evolves as a function of magnetic field strength, for different equations of state. Note that these curves correspond to structural parameters of that *particular* stellar configuration which *just* reaches the putative quark deconfinement density at zero angular velocity - we may call this the *critical configuration*. For

any given EoS, this is how we determine the minimum mass and spin period required for a quark phase to appear as a result of spindown. Unlike Fig. 4 of Staff et al. (2006), spindown is no longer always magnetic-dominated for arbitrary magnetic fields. For example, for $B \lesssim 10^{12}$ G, r-mode spindown takes over at a few hundred seconds. As the instability develops, magnetic damping F_m grows and limits the growth of the r-mode. This leads to a plateau in the period that can last up to several years before magnetic damping once again becomes the dominant driver of spindown. It is noteworthy that for the critical configuration, there is no change in the time to deconfinement for any value of B . However, if we choose a heavier mass at birth, the time to deconfinement would be shorter, and can occur while the star is still being spun down as a result of the r-mode. Assuming that even the most massive neutron stars can be transformed to stable quark stars if they reach the deconfinement density (that is, the quark matter EoS should be sufficiently stiff to support such a mass), we may expect that several such neutron stars would have already spun down to the point where they underwent a quark-hadron phase transition, and now contain quark matter in their core.

Our findings here are different from the conclusions of Ho & Lai (2000) who studied the r-mode of magnetized and slowly rotating neutron stars, and found that the r-mode alters the spindown if magnetic fields are less than $B \sim 10^{14}$ G. Due to the inclusion of the magnetic damping term, which was omitted in their work, we find that r-mode driven effects on spindown are pronounced only for $B \lesssim 10^{12}$ G, since for higher B fields, magnetic damping effectively kills the r-mode. We emphasize that the effects of magnetic damping for smaller B fields are not as severe as might be expected from the work of Rezzolla et al. (2001a) due to the self-consistent evolution of α in the presence of the magnetic damping term, as explained at the beginning of section 2.

We also find that the r-mode driven spindown is more pronounced in a relatively stiff equation of state such as EoS APR. This is because F_g , the gravitational damping rate is larger for a stiffer EoS², driving the r-mode unstable in a shorter amount of time (effectively spinning the star down to deconfinement density quickly).

The saturation amplitude α_{sat} of the r-mode assumed in the foregoing analysis is $\mathcal{O}(1)$. Such a large value is probably unrealistic due to multi-mode coupling and onset of non-linear effects (Arras et al. 2002; Bondarescu et al. 2008). If we use a smaller saturation value, we find that our conclusions change quantitatively. The r-mode grows more slowly for a smaller α_{sat} , but continues to be important since the main damping agent F_m , which is roughly proportional to α^4 , is also smaller. As a consequence, the spindown due to the r-mode is weaker, but also lasts longer, as seen in Fig.5. This can lead to the star taking a shorter time to reach a given period in the case of smaller B fields. This can be seen clearly in Fig.6 which compares the time taken by the star to spindown to a period $P=3$ ms, for $\alpha_{\text{sat}}=0.01$ and $\alpha_{\text{sat}}=0.005$.

² $F_g \propto MR^6/P^6$ and for stars with the same baryonic mass and central density at zero angular frequency, this quantity is systematically larger for a stiffer EoS. In other words, for a given baryonic mass, a stiffer EoS can support a larger gravitational mass and radius, and has a slightly smaller Kepler frequency since its average density is smaller (less compact). For eg., $F_g^{\text{APR}}/F_g^{\text{EoS A}} \approx 6$.

4. Evolution of the r-mode and Gravitational Wave Frequency

As shown in the previous section, fairly large magnetic fields of 10^{13}G or more are required to make the r-mode irrelevant for spindown. For smaller magnetic fields, the r-mode evolution can be the main agent for spinning down rapidly rotating stars. Following Owen et al. (1998), we can obtain expressions for the growth of the mode amplitude (α) and the corresponding evolution of the frequency of the gravitational wave (f) under certain approximations.

4.1. Saturated Phase

The r-mode amplitude grows rapidly until non-linear saturation occurs (Arras et al. 2002; Bondarescu et al. 2007) at which point $\dot{\alpha}=0$. We can obtain a description of this saturated phase from eqns.(4), which imply that the rotation rate evolves according to

$$\frac{\dot{\Omega}}{\Omega} = \frac{2\alpha^2 K_c F_g - F_{\text{mag}}}{1 - \alpha^2 K_c (1 - K_j)}, \quad (6)$$

where $\alpha = \alpha_{\text{sat}}$ is the saturation amplitude of the r-mode. Note that F_m does not appear since it can be eliminated using the second line of eqns.(4). Its effect will however show up in the growth phase discussed below. Using the fact that $\Omega = 3\pi f/2$ for the $l = m = 2$ r-mode, setting $K_c = 0.1$ (a typical value determined from eqn.(2)), and using the frequency-dependent expression for the gravitational wave damping timescale $\tau_{\text{GR}} \equiv F_g^{-1}$, we find

$$\frac{\dot{f}}{\text{Hz}^2} \approx -0.35\alpha^2 \left(\frac{f}{\text{kHz}}\right)^7 - 0.0055 \frac{B_{14}^2 R_6^6}{I_{45}} \left(\frac{f}{\text{kHz}}\right)^3, \quad (7)$$

where we have used the expression for τ_{GR} from Lindblom et al. (1998) and neglected $\alpha^2 K_c (1 - K_j) \ll 1$ in the denominator of eqn.(6). B , R and I are expressed in terms of reduced dimensionless units.

4.2. Growth Phase

In the initial stages of the mode growth phase, the viscosity controls the evolution of the r-mode, but the instability to gravitational waves soon takes over. Viscous damping from bulk viscosity can be large at $T \geq 10^{11}\text{K}$, but as the neutron star cools rapidly on the order of seconds, our approximation of setting $T_9 = 1$ implies that viscosity does not affect the subsequent spindown behaviour or the gravitational waveform during the growth of the r-mode instability. It follows from eqns.(4) that during this phase, since α is small

$$\frac{\dot{\alpha}}{\alpha} = (F_g - F_m) - \frac{\dot{\Omega}}{2\Omega}. \quad (8)$$

The angular velocity evolves according to the first line of eqns.(4). We assume $K_j \approx 0$ for the growth phase, which is tantamount to including fully the canonical angular momentum of the r-mode in the star's physical angular momentum. This is only justified if differential rotation is small (Sa & Tome 2005), as is the case when the mode is still small but growing. Then, as before, the proportionality between f and Ω implies

$$\frac{\dot{f}}{\text{Hz}^2} \approx -0.0131\alpha^2(t) \left(\frac{f}{1\text{kHz}} \right)^3 - 0.0055 \frac{B_{14}^2 R_6^6}{I_{45}} \left(\frac{f}{1\text{kHz}} \right)^3 - \frac{0.202 B_{14}^2 R_6}{\bar{J} M_{1.4}} \alpha^2(t) \int_0^t \alpha^2(t') \left(\frac{f(t')}{\text{Hz}} \right) dt'. \quad (9)$$

where we have used the expression for F_m from eqn.(11) of Cuofano & Drago (2010). We can now use eqns.(7) and (9) to obtain the gravitational strain amplitudes and waveforms for the saturated phase and growth phase respectively.

5. Results: Gravitational Strain Amplitudes and Waveforms

The strain amplitude $h(t)$ corresponding to the $l=m=2$ mode is found by a standard multipole analysis (Thorne 1980). Including the angle-average over the position of sources in the sky,

$$h(t) = \sqrt{\frac{3}{80\pi}} \frac{\omega^2 S_{22}}{D}, \quad (10)$$

$$S_{22} = \sqrt{2} \frac{32\pi}{15} \frac{GM}{c^5} \alpha \Omega R^3 \bar{J}. \quad (11)$$

where S_{22} is the current multipole as given by eqn.(3.9) of Owen et al. (1998), the mode frequency $\omega = 4\Omega/3$ and D is the source distance, chosen henceforth to have a typical value of 20 Mpc³. Figure 2 shows the time-evolution of the strain amplitude $h(t)$ for various magnetic fields and for different EoS for $\alpha_{\text{sat}} = 0.5$, while Fig. 7 shows this for $\alpha_{\text{sat}} = 0.01$. For magnetic fields $B \geq 10^{14}\text{G}$, the r-mode withers much before it nears saturation, for any EoS - the strain amplitude is consequently very small. For $B \sim 10^{13} - 10^{14}\text{G}$, we notice a strong dependence on the equation of state. The soft EoS A does not lead to a saturating r-mode while relatively stiffer EoS do. For weaker magnetic fields around $B \sim 10^{12}\text{G}$, the r-mode saturates and displays the behaviour shown in Fig. 5 of Owen et al. (1998).

We now examine the gravitational waveform in the frequency domain $\tilde{h}(f)$ which is the Fourier transform of the time signal. This is useful in estimating the signal-to-noise ratio (SNR) in matched filtering techniques. In the stationary phase approximation,

³This fiducial value of D extending to the Virgo cluster is chosen to encompass enough neutron stars to ensure a reasonable event rate (Owen et al. 1998).

$$|\tilde{h}(f)| = \sqrt{\frac{|h(t)|^2}{|\dot{f}|}}. \quad (12)$$

From eqns.(7), (9) and (10), we can find $\tilde{h}(f)$ for both the growth phase and the saturated phase. From Figs. 3 and 8 (for $\alpha_{\text{sat}} = 0.5$ and 0.01 respectively), we see that for magnetic fields $B \leq 10^{13}\text{G}$, the signal from the r-mode is similar to previous result obtained in the absence of magnetic fields (Owen et al. 1998). A sharp peak at high frequency (≈ 1.6 kHz) corresponding to the growth phase is seen, followed by a plateau signal at lower frequencies as the r-mode saturates. Note that the curves terminate on the left since at that point, deconfinement density is reached and the subsequent signal from the phase transition needs a detailed analysis beyond the scope of this work. However, we refer the reader to the Appendix for a preliminary treatment of this issue. Previous works (Schenk et al. 2002; Morsink 2002; Brink et al. 2004; Bondarescu et al. 2007) indicate that the saturation amplitude α_{sat} may be quite small, around 10^{-2} instead of order 1. Still, it is useful to contrast both cases: when $\alpha_{\text{sat}} \approx 0.5$ and $\alpha_{\text{sat}} \approx 0.01$.

5.1. Signal Detectability

In Fig. 4 we present our estimate of the continuous gravitational wave signal from the spin-down (assuming a point source at a distance of 20 Mpc), compared to the anticipated sensitivity of Advanced LIGO ⁴ and the Einstein telescope (Hild et al. 2010). The left panel of Fig. 4 shows the weighted strain amplitude ($1/\sqrt{\text{Hz}}$) versus frequency for the three EoS studied in this paper at 10^{13} G, compared to the anticipated noise-weighted design sensitivities of Advanced LIGO and the Einstein telescope. The right panel shows the same for 10^{14} G. In both figures, we are assuming $\alpha_{\text{sat}} \approx 0.01$. The saturated phase of the curve is at least an order of magnitude above the anticipated sensitivity for Advanced LIGO for all EoS at 10^{13} G and could therefore be expected to be observed out to 20 Mpc with Advanced LIGO, with a typical SNR of ~ 20 . At 10^{14} G and for the softest EoS, the SNR in Advanced LIGO drops to ~ 2 . The Einstein telescope is expected to gain about an order of magnitude in sensitivity compared to Advanced LIGO, and hence all of the above cases should be detectable in principle. Our SNR estimates only provide upper limits since matched filtering is probably impractical for such sources, where spin parameters cannot be promptly measured, though the source location could be known if the supernova is observed. However, we note that most of the signal comes from the saturated phase, which for the case of small α_{sat} occurs several years after a neutron star's birth in a supernova. Observations of the neutron star's spindown parameters may then be possible, making the matched filtering method more feasible. In the context of r-modes in newly-born neutron stars, methods other than matched filtering

⁴These curves represent the incoherent sum of the principal noise sources, such as quantum noise, seismic noise and thermal noise, as best understood at this time. There will be, in addition, technical noise sources. These curves serve as a guide to the overall curve and an early approximation to the anticipated sensitivity (LIGO Scientific Collaboration 2010).

have been suggested (Brady & Creighton 1998; Owen et al. 2010; Zhu et al. 2011) that would be less computationally intensive.

5.2. Saturation at $\alpha_{\text{sat}} \approx 0.5$

As mentioned in Owen et al. (1998), the sharp spike in the strain amplitude seen in certain cases only lasts for a brief period of time (of the order of a minute) and carries a small fraction of the total energy emitted in gravitational waves over the spindown epoch (see Fig. 2). However, for the softer EoS, such as BBB2, we see that for magnetic fields $B \approx 10^{15}$ G, the spike is softened into an asymmetric hump. In this case, the growth phase lasts about an hour and carries a larger fraction of the emitted energy - it may be detectable owing to its larger SNR in second and third generation detectors. Furthermore, for the softest EoS considered here (EoS A), there is no saturation regime for fields larger than $B \approx 10^{12}$ G. These are potentially distinguishing features of the equation of state in gravitational waves. In general, for very high magnetic fields $B \geq 10^{15}$ G, the r-mode is strongly suppressed, and the signal in gravitational waves is weak, with little possibility of detection even in third generation detectors.

5.3. Saturation at $\alpha_{\text{sat}} \approx 0.01$

For a more realistic value of α_{sat} , we find that the strain amplitude is about an order of magnitude smaller than for $\alpha_{\text{sat}}=0.5$, but the signal persists for a much longer time (several months/years for $B \leq 10^{14}$ G). This is clear from Fig. 7. This is because of the dependence of F_m on B^2 and $\alpha(t)$. For smaller B fields and α_{sat} , the weaker magnetic damping allows the r-mode to survive for a longer time, even though mode growth is slower. This eventually leads to a *faster* spindown for the star, compared to a higher α_{sat} . To emphasize this effect, in Fig. 6, we compare spindown with $\alpha_{\text{sat}}=0.01$ and 0.005, for the same initial stellar configuration and the same magnetic field. The crossing of the two curves in that figure illustrates this effect. However, we note from comparing Figs. 3 and 8 that \tilde{h} remains relatively independent of α_{sat} , although both the $B = 10^{13}$ G and $B = 10^{14}$ G cases show a flat segment corresponding to the saturated phase for $\alpha_{\text{sat}} = 0.01$. Essentially, this is because the decrease in the peak value of $h(t)$ for smaller α_{sat} is compensated by the corresponding decrease in \dot{f} (eg., as in eqn.(7)). This implies that we require a larger interval of time integration to validate the stationary phase approximation for a smaller α_{sat} (eqn. 12). Thus, \tilde{h} remains almost unchanged.

6. Conclusions

We have studied the role of r-modes in spinning a rapidly rotating and magnetized neutron star down to typical quark deconfinement densities. Apart from the usual spindown associated to magnetic braking, the magnetic damping of r-modes plays an important role in the period evolution

of the star. We find that, for realistic (small) values of the r-mode saturation amplitude α_{sat} , the time to deconfinement is sped up by the r-mode instability when magnetic fields are of order 10^{12}G or less. This result reflects the strong dependence of the magnetic damping effect on B and the evolution of α . Thus, in contrast to the result in (Ho & Lai 2000), where the r-mode was seen to affect the spindown already for magnetic fields less than $B \sim 10^{14}\text{G}$, the inclusion of the magnetic damping term leads us to conclude that r-mode driven effects on spindown are pronounced only for $B \lesssim 10^{12}\text{G}$. We also explored the gravitational wave signal generated during the spindown phase as the r-mode first grows then saturates - we follow the signal continuously until the quark deconfinement threshold is reached. For realistic values of α_{sat} , the r-mode saturates and leads to a strong signal in gravitational waves, except in case of very soft equations of state and very large magnetic field (10^{15} G or more). There could be a sizeable fraction of the total rotational kinetic energy emitted as gravitational waves in this epoch, which can last several years, making it detectable in upcoming second and third generation detectors. Therefore, gravitational waves could be used in this manner to probe the equation of state inside neutron stars. As shown here, using r-modes is an alternate way in which the EoS can be probed through gravitational waves ⁵

Gravitational waves are going to open a new window of observation into our universe. Among the many discoveries that will be made, we anticipate the exciting prospect that neutron star spindown will reveal signs of the elusive r-mode instability as well as signatures for the onset of quark matter in the core. In the appendix we outline a first preliminary estimate of what the gravitational wave signal from a “Quark-Nova” would look like. It would be interesting to examine this signal with detailed simulations, especially in cases when the Quark-Nova occurs very shortly after the neutron star is born in a Supernova - the signature of this ”dual-explosion” in gravitational wave detectors would be two very different signals coming from the same source but separated in time by a few days to few weeks, depending on the time delay between the two explosions. Such a signature would be unmistakable in upcoming gravitational wave detectors.

Appendix: Gravitational waves from the quark-hadron phase transition

It remains an open question as to what happens to the neutron star when the deconfinement density is reached. One possibility is that the entire star is converted to a quark star due to the inherent stability of strange quark matter (Witten 1984). If this conversion occurs in an explosive manner, a “Quark-Nova” could result (Ouyed et al. 2002) - what will the gravitational signal from such an event look like? The conversion involves a two-stage process - neutrons in the core dissolve into a u and d quark fluid that is more compact, causing the core to shrink, followed by combustion to u, d, s quarks through leptonic as well as non-leptonic processes. Lin et al. (2006) have studied

⁵We note that recently derived empirical upper bounds on the gravitational power radiated by the Crab pulsar constrain the ellipticity of deformed pulsars (Pitkin 2011; Abbott et al. 2008, 2010) with implications for the maximum theoretical elasticity of the neutron star crust (Horowitz & Kadau 2009). On the other hand, in order to similarly constrain the r-mode amplitude from searches directed at known pulsars, a different range of frequencies and polarizations must be probed (Owen et al. 2010), thus the limits placed from the Crab pulsar do not have direct bearing on r-modes.

the first stage with Newtonian hydrodynamics and found that quadrupolar and quasi-radial modes are excited during the collapse, leading to gravitational wave emission with an energy output of $\sim 10^{51}$ ergs. However, this work applies to a mixed phase of quarks and nuclear matter and does not consider an explosive transition that begins with non-premixed fluids.

A full numerical treatment of the second stage, an explosive phase transition taking into account fluid motion in 3D is a complex task and beyond the scope of this work. However, preliminary steps have been taken in this direction. Niebergal et al. (2010) solved hydrodynamical flow equations for the combustion of neutron matter to strange quark matter in the laminar approximation, including weak equilibrating reactions and strange quark diffusion across the burning front. The numerical results suggest laminar speeds of 0.002 – 0.04 times the speed of light, much faster than previous estimates derived using only a reactive-diffusive description (Olinto 1987). Turbulent combustion has been addressed in a recent work by Herzog & Roepke (2011) who found that the combustion stops short of converting the entire star to quark matter (the reaction is no longer exothermic). This was also the conclusion in Niebergal et al. (2010), though for a different physical reason⁶. Neither of these works continue on to estimate the gravitational signal from the explosive combustion. Following the hydrodynamical approach of these studies, we have estimated the gravitational wave signal $h(t)$ from the following steps.

- Starting from a mechanically stable configuration where quark matter constitutes a small fraction of the stellar core, we initiate combustion at the speeds obtained in Niebergal et al. (2010). The quark fluid, described by a simple bag model equation of state is subsequently evolved using inviscid hydrodynamical equations for relativistic fluid flow (relativistic Euler equations) coupled with Newtonian gravity⁷, assuming an axisymmetric rotating configuration of the star about the \hat{z} -axis. In the cylindrical coordinates (r, ϕ, θ) , where ϕ is the polar angle and θ is the azimuthal angle, the relevant equations are given by $\partial_t(rU) + \partial_r(rF) + \partial_z(rG) = S'$, where

$$U = \begin{pmatrix} D \\ Su_r \\ Su_\theta \\ Su_z \\ \tau \end{pmatrix} \quad F = \begin{pmatrix} Du_r \\ Su_r^2 + p \\ Su_\theta u_r \\ Su_z u_r \\ (\tau + p)u_r \end{pmatrix} \quad G = \begin{pmatrix} Du_z \\ Su_r u_z \\ Su_\theta u_z \\ Su_z^2 + p \\ (\tau + p)u_z \end{pmatrix} \quad S' = \begin{pmatrix} 0 \\ Su_\theta^2 + p + rDg_r \\ -Su_\theta u_r \\ rDg_z \\ rS(u_r g_r + u_z g_z) \end{pmatrix}$$

and where $D = \rho\gamma$, $S = Dh\gamma$, and $\tau = S - D - p$ are introduced solely for the purpose of writing the relativistic Euler equations in a form analogous to the more familiar non-relativistic Euler

⁶Niebergal et al. (2010) found that as the burning front expands and cools, it enters an advection dominated regime, where the upstream (hadronic) fluid velocity advects the interface backwards faster than it can progress due to reactions and diffusion. Consequently, the interface halts short of the neutron star surface.

⁷ Although the Poisson equation for gravity violates the speed of light (since it is an elliptic PDE), it is unavoidable unless one uses the full GR.

equations. In the natural units where $c = 1$, the fluid velocity, Lorentz factor, gravitational vector field, and specific enthalpy are respectively given by $\vec{u} \equiv (u_r, u_\theta, u_z)$, $\gamma = 1/\sqrt{1 - |\vec{u}|^2}$, $\vec{g} = -\nabla\phi$, and $h = 1 + \epsilon + p/\rho$, where ϵ is the specific internal energy, p is the pressure, and ϕ is the gravitational potential. In the non-relativistic limit ($\gamma \rightarrow 1$), we recover the non-relativistic Euler equations.

- The quark matter has a density $\rho_q = 5\rho_n$ and is described by the Bag model (bag constant $B^{1/4} = 145$ MeV) while the nuclear fluid has density $\rho_n = \rho_{\text{sat}} = 2.5 \times 10^{14} \text{ g/cm}^3$ and is described by the perfect caloric EoS with an adiabatic index of 1.7. At $t=0$, we initiate combustion with a density discontinuity $\rho_q - \rho_n$, which is situated at a radial coordinate $r = R/4$ inside the star, and choose an initial burning front speed of $u_r=0.01c$, which is in the range of burn velocities obtained in Niebergal et al. (2010). We then evolve the above equations using a weighted average flux relativistic HLL (Harten, Lax, and Van Leer) solver on a cartesian grid with spatial resolution 0.5 km and a timestep determined by a Courant number of 0.3. The solver is coupled with the Poisson equation $\nabla^2\phi = 4\pi G\rho$.
- Solving for $\vec{u}(r, z, t)$ and $\rho(r, z, t)$, we follow Zwerger & Muller (1997) to compute the quadrupole wave amplitude $A_{20}^{E2}(t)$ for our axisymmetric configuration, and from there the signal $h(t)$ as given by eqn.(21) of (Zwerger & Muller 1997).

$$h(t) = \frac{1}{8} \sqrt{\frac{15}{\pi}} \sin^2\theta \frac{A_{20}^{E2}(t)}{D} \quad (1)$$

We assume a source distance $D = 20$ Mpc to plot the maximal signal strength $h(t)$ and the corresponding luminosity $L(t)$ in Fig.9. The origin of the peak in $h(t)$ is a balance between increasing mass outflow and a decreasing density gradient. The luminosity is proportional to the square of $\dot{h}(t)$ (hence the sharp dip at the maximum of $h(t)$) while the total emitted energy obtained from integrating the luminosity curve is $\sim 1.4 \times 10^{48}$ ergs, which is about 5 orders of magnitude smaller than the binding energy of the neutron star (10^{53} ergs) and 2 orders of magnitude less than the energy released in gravitational waves during the first stage of core-collapse, where the energy comes from coupling oscillations to rotational motion (Lin et al. 2006; Marranghello et al. 2002).

Based on this result, the gravitational wave signal for this stage of the phase conversion would be hard to detect in Advanced LIGO, unless the source is Galactic (within few kpc). Most of the energy released in the phase transition is in the form of latent heat and neutrinos. However, our results are only a preliminary estimate, designed to provide a guide to the expected signal from a Quark-Nova. Simulations of gravitational wave signals from realistic core-collapse supernova models indicate that convective flows driven by neutrino heating (Müller et al. 2004) can drive a strong gravitational wave signal. The typical time for neutrinos to diffuse out of the hot quark star (≈ 0.1 -1 sec according to Keranen et al. 2004) is about an order of magnitude larger than the duration of the signal due to explosive combustion to strange quark matter, therefore, we can expect neutrino heating to be important in our context as well. We have not included such convective effects in

our present simulation, nor the effect of magnetic fields, which can also modify Rayleigh-Taylor instabilities (Lugones et al. 2002).

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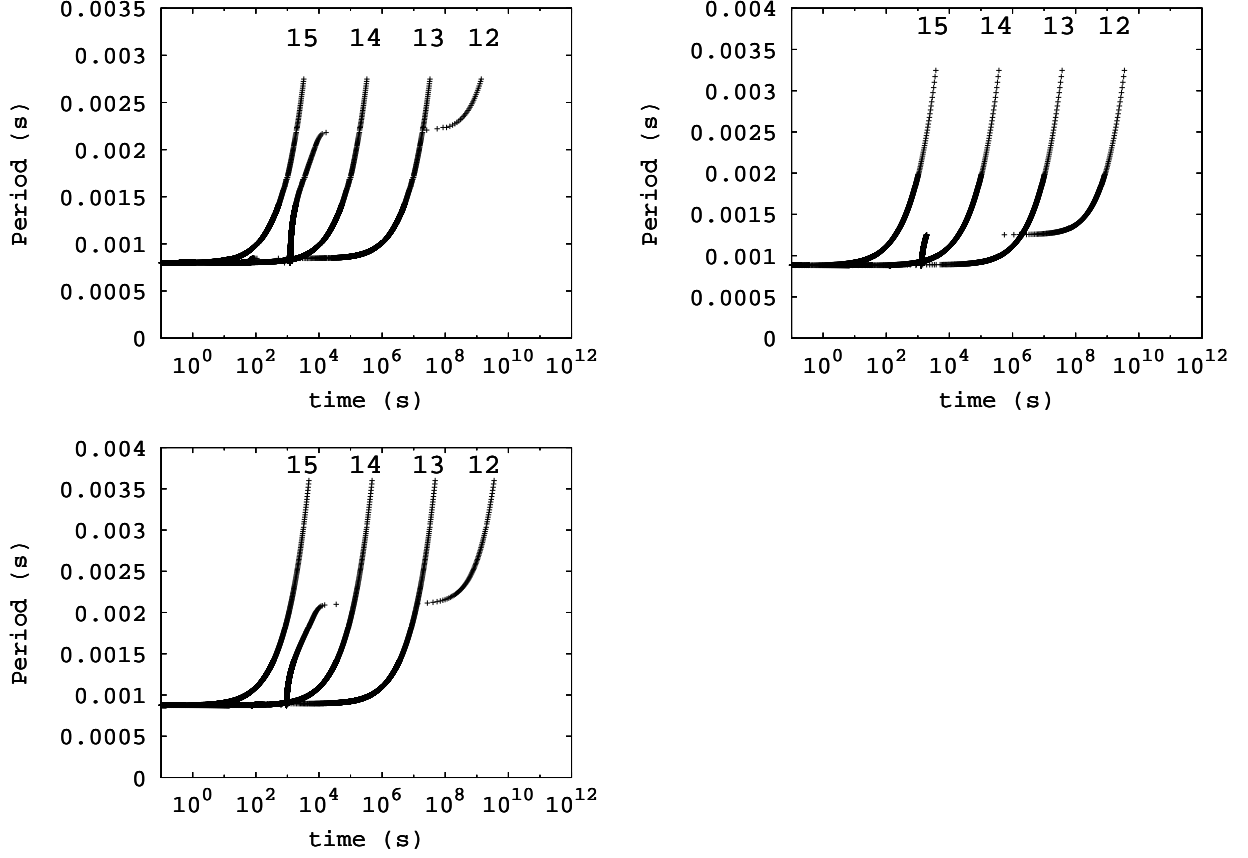


Fig. 1.— Period vs time for the APR EoS (*top left*), EoS A (*top right*), and EoS BBB2 (*bottom left*) with a constant temperature of $T = 10^9$ K. The curves from left to right are labelled for $\log(B)$. For $B \geq 10^{13}G$, magnetic braking starts to dominate the r-mode as far as spindown is concerned. The curves terminate abruptly at the moment when the central density is within 1% of the critical density (see discussion above eqn.(5)).

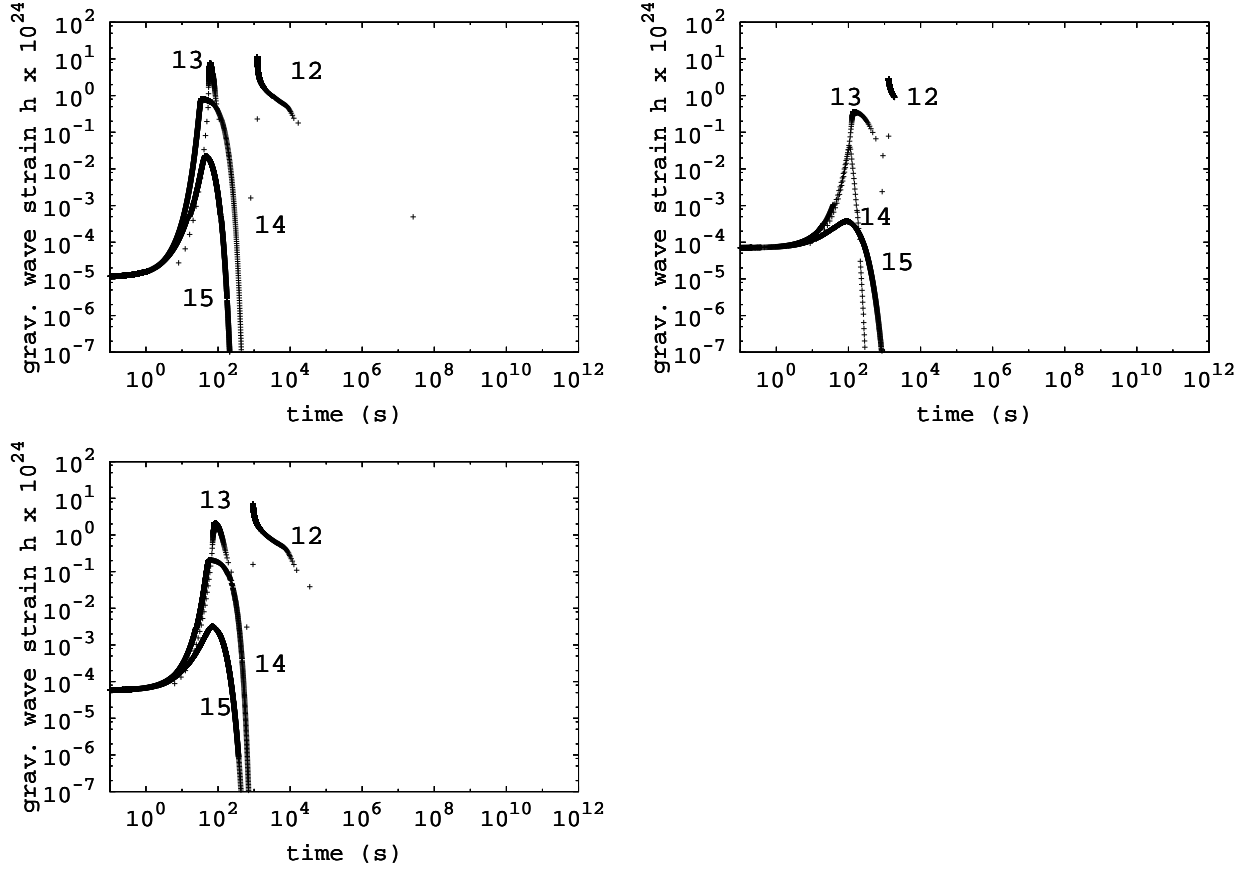


Fig. 2.— Gravitational wave strain as a function of time for the APR EoS (*top left*), EoS A (*top right*), and EoS BBB2 (*bottom left*) with a constant temperature of $T = 10^9$ K. The curves for softer EoS and larger B fields have a much lower peak strain as in these cases the r-mode amplitude never saturates.

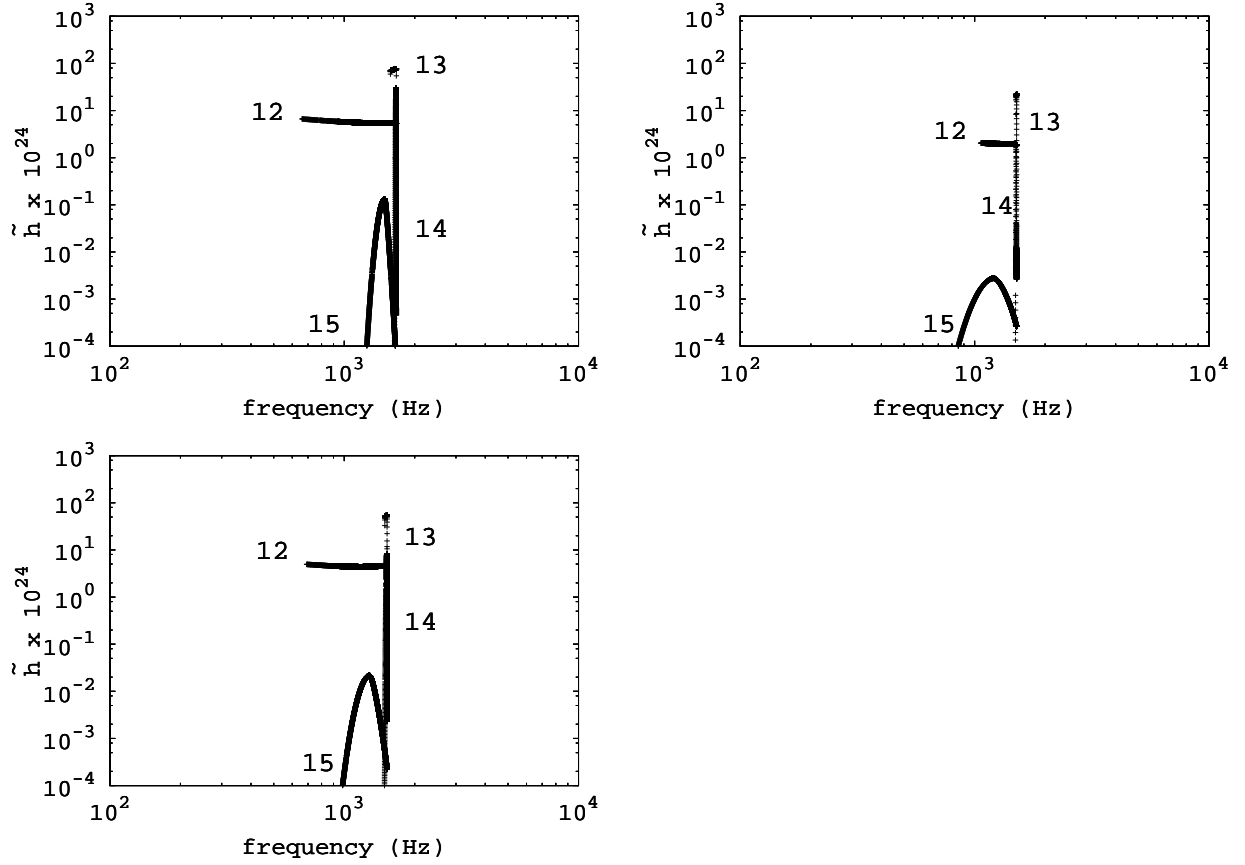


Fig. 3.— Gravitational wave strain as a function of frequency for the APR EoS (*top left*), EoS A (*top right*), and EoS BBB2 (*bottom left*) with a constant temperature of $T = 10^9$ K. The 10^{14} G and 10^{15} G curves are distinct from the rest, as in these cases the r-mode amplitude barely or never saturates.

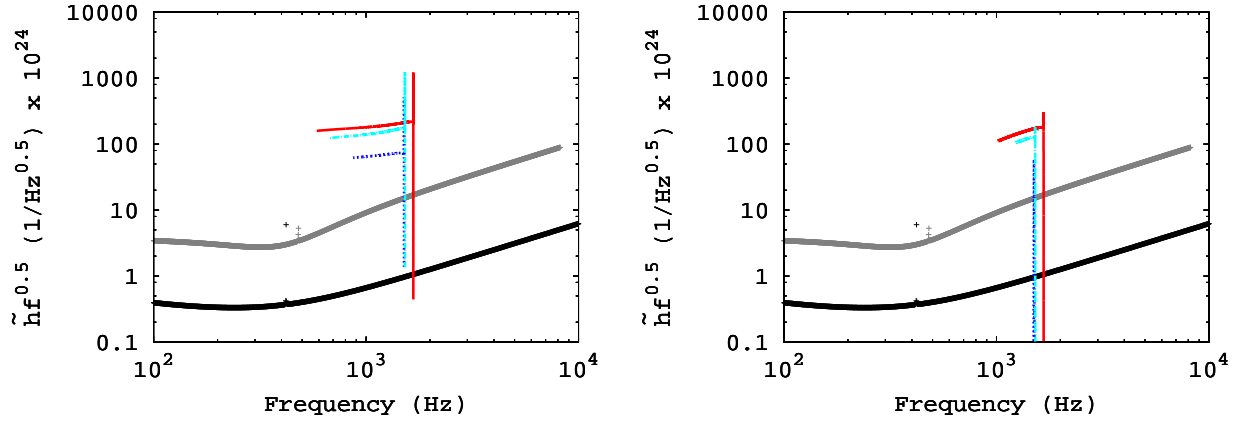


Fig. 4.— Gravitational wave strain as a function of frequency for three different EoS - APR(solid), BBB2(dash-dotted) and EoS A(dotted), compared against sensitivity curves (strain noise) for (grey) Advanced LIGO taken from Harry et al. (2010) for the LIGO Scientific Collaboration, and (black) the Einstein Telescope from Hild et al. (2010). Left panel is with $B=10^{13}$ G and right panel with $B=10^{14}$ G. The saturation amplitude $\alpha_{\text{sat}}=0.01$.

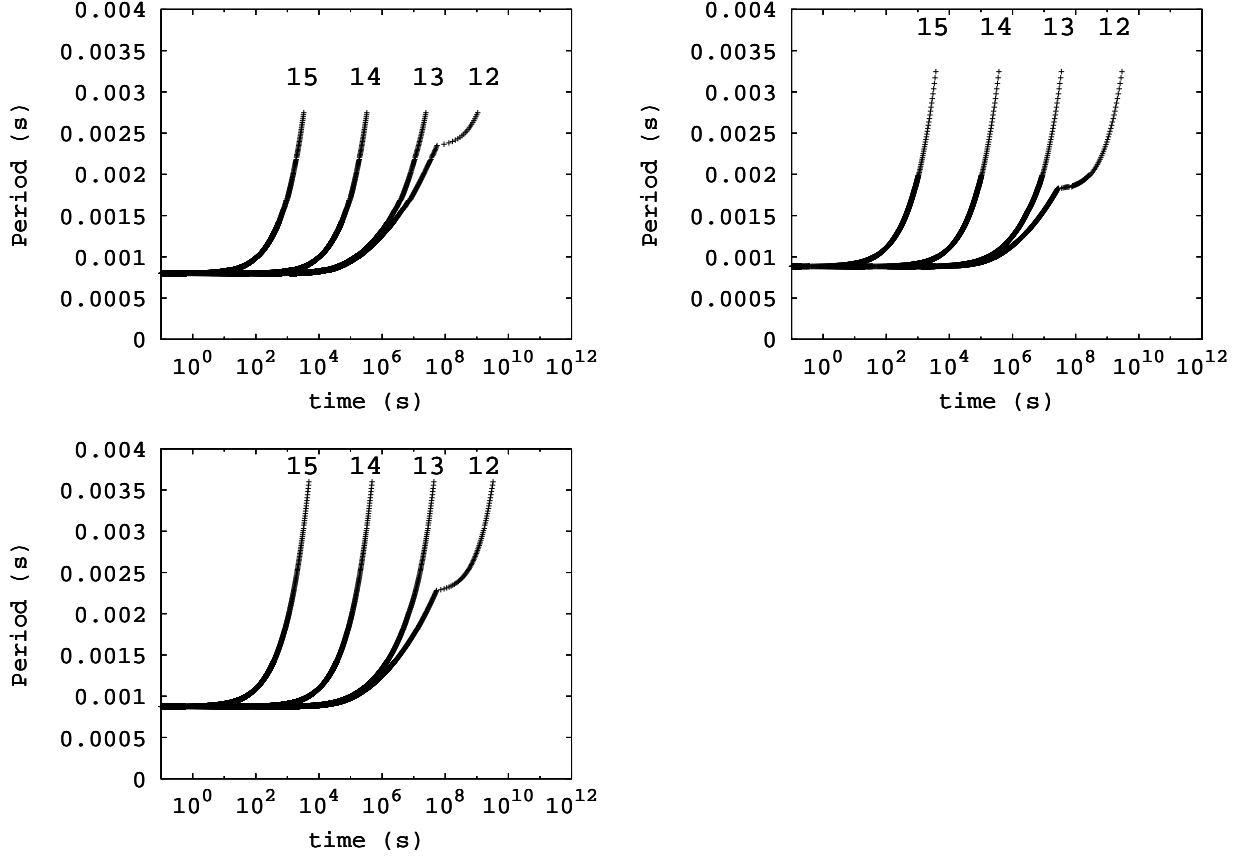


Fig. 5.— Same as Fig. 1, but with r-mode saturation value $\alpha_{\text{sat}} = 0.01$. Note that the effect of the r-mode is still significant but appears later than for $\alpha_{\text{sat}} = 0.5$. This is due to the nature of magnetic damping.

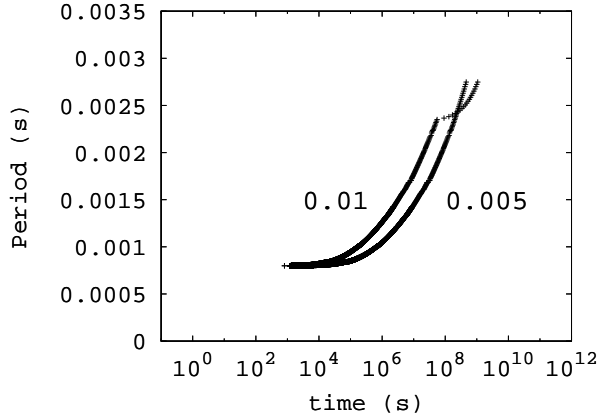


Fig. 6.— Spindown from Kepler frequency for identical stellar configurations (APR EoS with $B=10^{12}$ G) but with different r-mode saturation values: $\alpha_{\text{sat}}=0.01$ and 0.005 . Time to reach $P=3$ ms (where supposed deconfinement density $5\rho_0$ is reached) is shorter for smaller α_{sat} .

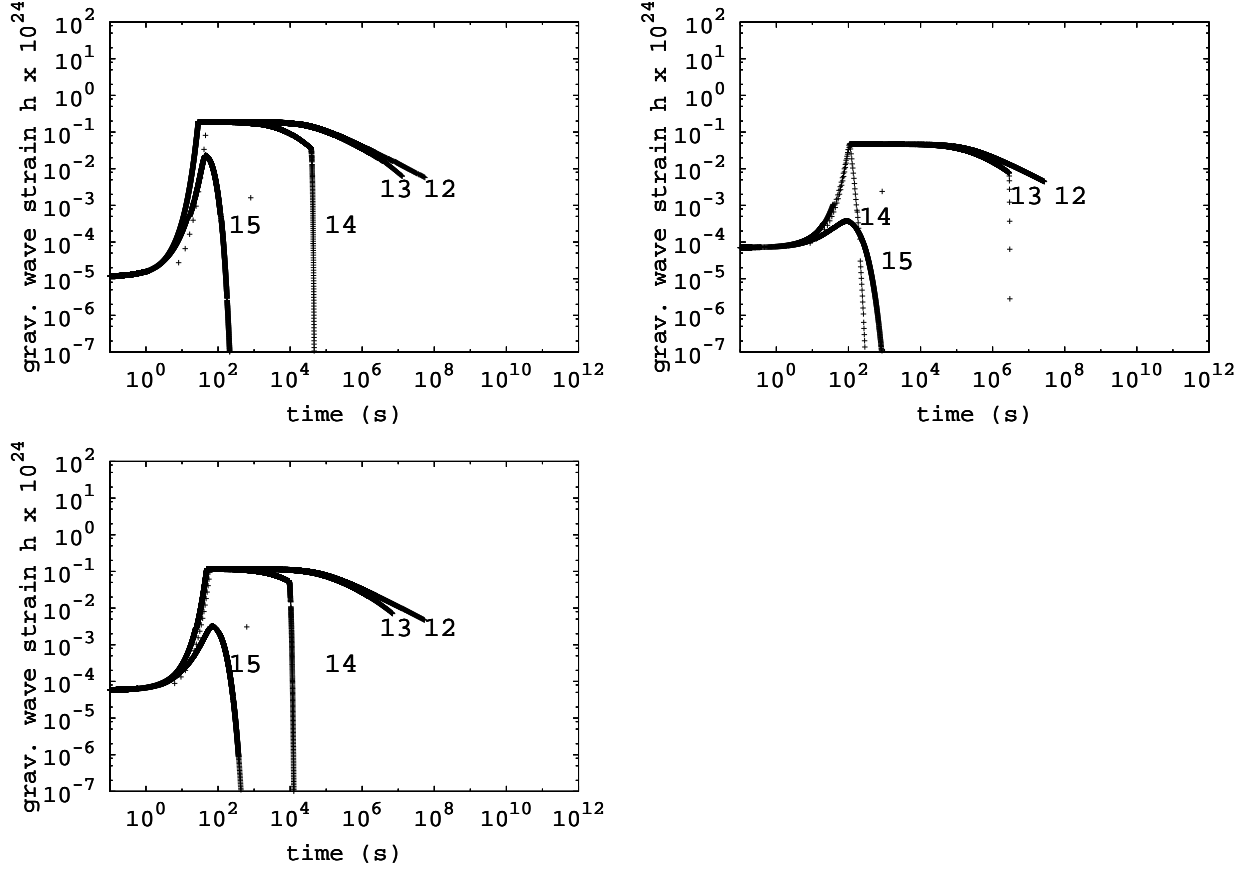


Fig. 7.— Same as Fig. 2 but with $\alpha_{\text{sat}} = 0.01$. For smaller values of B , the signal reflects the fact that the r-mode evolves on much longer timescales than for $\alpha_{\text{sat}} = 0.5$.

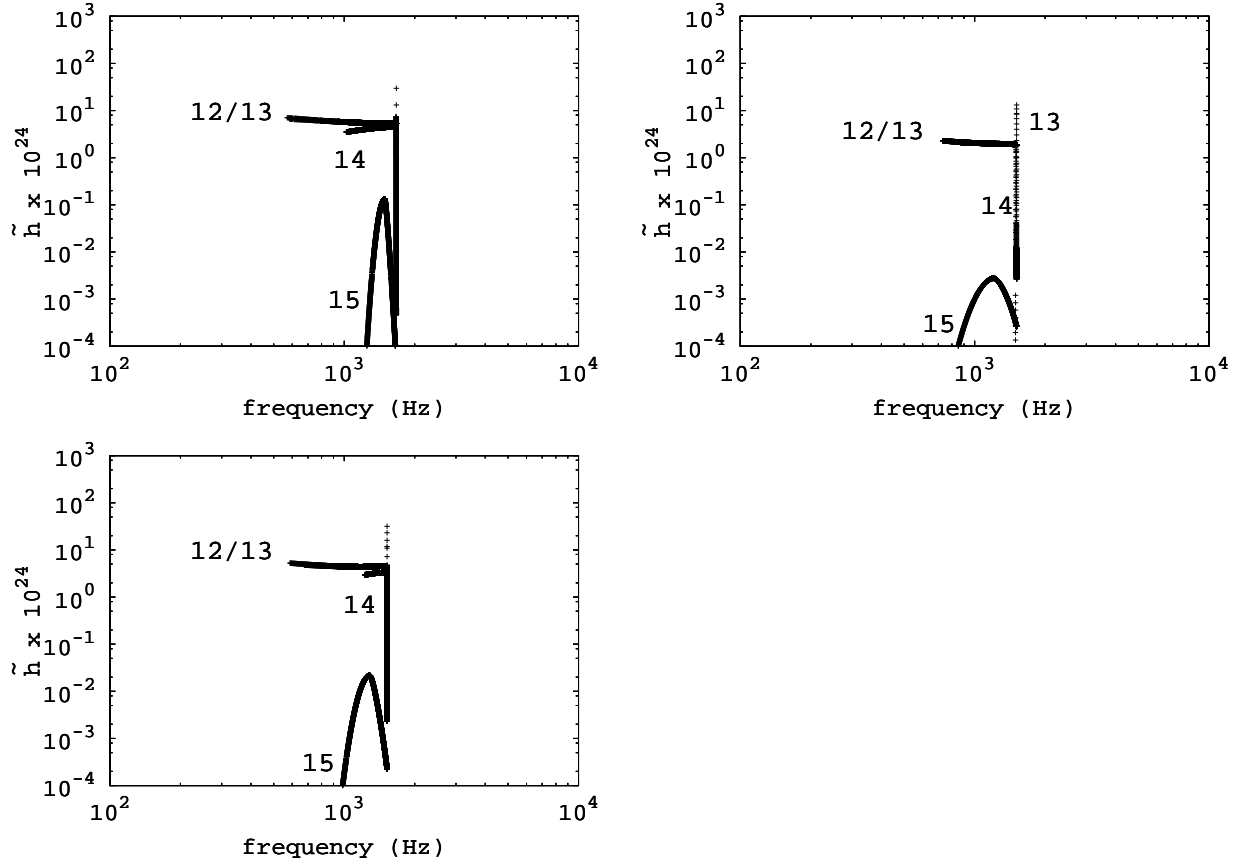


Fig. 8.— Same as Fig. 3, but with $\alpha_{\text{sat}} = 0.01$. The 10^{15} G (and for EoS A also the 10^{14} G) are distinguished, as in these cases the r-mode amplitude never saturates even when we assume such a low saturation value.

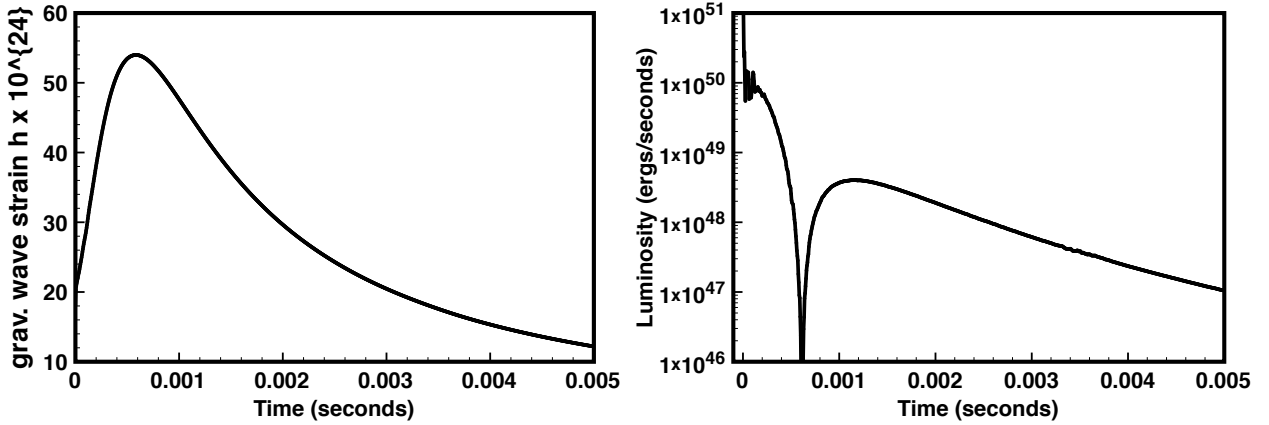


Fig. 9.— Gravitational wave strain $h(t)$ and luminosity $L(t)$ as a function of time for combustion of neutron matter to strange quark matter following quark deconfinement inside a neutron star.